Lesson 28

Math 70 Chapter 11 Quadratics

11.1 Solving Quadratic Equations by Completing the Square
11.2 Solving Quadratic Equations by the Quadratic Formula
11.3 Identifying the type and number of solutions using the Discriminant
11.4 Applications of Quadratic Equations
11.5 Solving Equations that are Quadratic in Form (or "Reducible to Quadratic")

We will finish chapter 11 when we graph parabolas of all types in chapter 13, including the following:
11.6 Graphing Quadratic Functions
11.7 Graphing Quadratic Functions, Vertex Formula, Maximum or Minimum Value
11.8 Applications of Quadratic Functions

<u>11.1 Solving Quadratic Equations by Completing the Square</u>

Objectives:
1) Use the square root property to solve quadratic equations where all the variables are within an expression.
2) Solve quadratic equations by completing the square, then using the square root property.
3) Check solutions by looking at graphs on GC.
4) Check solutions by evaluating the function on the GC.

<u>Warm-ups</u>

Simplify:

1) $\frac{9}{10}-\sqrt{\frac{-361}{100}}$ 2) $\frac{12-8\sqrt{7}}{16}$ 3) $\frac{10-20\sqrt{-288}}{16}$

Important: "Solve" means find *all* values of the variable which make the equation true. When using the square root property, we must remember the $\pm$, because without it we are missing a solution.

4) Solve by factoring, then by the square root property: $x^2-81=0$

Solve by the square root property.

5) $2x^2-24=0$

6) $(2x-5)^2=-16$

7) $3p^2-6p=4$
   a. Solve by CTS & square root property
   b. Identify the number and type of solutions using the previous work.
   c. Check solutions by evaluating.

Math 70

(1) $\frac{9}{10} - \sqrt{\frac{-361}{100}}$

$= \frac{9}{10} - \frac{\sqrt{-361}}{\sqrt{100}}$ quotient property of square roots

$= \frac{9}{10} - \frac{\sqrt{-1} \cdot \sqrt{361}}{\sqrt{100}}$ product property of square roots

$= \frac{9}{10} - \frac{i \quad 19}{10}$

$19^2 = 361$
$10^2 = 100$
$i = \sqrt{-1}$ by definition

$= \boxed{\frac{9}{10} - \frac{19}{10}i}$

$\uparrow \qquad \uparrow$
$a + b \cdot i$

Standard form for complex numbers is a+bi form, separate numbers for real part a and imaginary part b.

CAUTION: MML/MXL will accept $\frac{9-19i}{10}$ even though this is nonstandard notation.

Do NOT do this on a PQ or Exam!

(2) $\frac{12 - 8\sqrt{7}}{16}$

$= \frac{12}{16} - \frac{8\sqrt{7}}{16}$ divide each term by denominator

$= \boxed{\frac{3}{4} - \frac{\sqrt{7}}{16}}$

Option 2: Factor numerator

$\frac{4(3 - 2\sqrt{7})}{16}$

$= \boxed{\frac{3 - 2\sqrt{7}}{4}}$

Note: Neither contains i, so not a complex number. Either answer is acceptable.

Math 70

(3) $\dfrac{10 - 20\sqrt{-288}}{16}$

Several tasks must be completed, but can be done in any order

1. simplify $\sqrt{-1}$
2. simplify $\sqrt{288}$
3. Because $\sqrt{-1} = i$, must be $a+bi$ form. Divide both terms by 16.

$= \dfrac{10}{16} - \dfrac{20}{16} i\sqrt{288}$

simplify $\sqrt{-1} = i$
divide both terms by 16

$= \dfrac{5}{8} - \dfrac{5}{4} i \cdot \sqrt{144 \cdot 2}$

$$\begin{array}{ccc} & 288 & \\ & \wedge & \\ 144 & & 2 \end{array}$$

prime factors of 288
OR
divide by the biggest perfect square.

4	49
9	64
16	81
25	100
36	121
	144

$= \dfrac{5}{8} - \dfrac{5}{4} i\sqrt{144} \cdot \sqrt{2}$

product property of radicals

$= \dfrac{5}{8} - \dfrac{5}{4} i (12)\sqrt{2}$

reduce $\dfrac{12}{4} = 3$

$= \dfrac{5}{8} - 5 \cdot 3 i\sqrt{2}$

$= \boxed{\dfrac{5}{8} - 15 i\sqrt{2}}$ or $\boxed{\dfrac{5}{8} - 15\sqrt{2}\, i}$

Caution: If using this format, i must be clearly outside $\sqrt{\ }$

$\sqrt{i}$ is not what you mean, and it does mean something else.

$\sqrt{i} = \sqrt{\sqrt{-1}} = \sqrt[4]{-1}$

Math 70

To solve by square root property

step 1: Isolate the square.

step 2: Use square root property take $\sqrt{\ }$ both sides, using $\pm$

step 3: simplify the square root

step 4: Isolate the variable if necessary.

step 5: Write solutions using 2 separate fractions. to get $a+bi$ form.

Math 70

(4) $x^2 - 81 = 0$

Solve by factoring the difference of squares

$(x-9)(x+9) = 0$

$\boxed{x = 9 \quad x = -9}$

There are two solutions. Any correct method must find both.

Solve by square root property

<u>step 1</u>: Isolate the perfect square.

<u>step 2</u>: Square root both sides

<u>step 3</u>: Include $\pm$ to capture both solutions

$x^2 - 81 = 0$

$x^2 = 81$ isolate square = get x^2 alone

$\sqrt{x^2} = \sqrt{81}$

$|x| = 9$ technically, $\sqrt{x^2} = |x|$, which is where the $\pm$ comes from!

$\boxed{x = \pm 9}$

(5) $2x^2 - 24 = 0$

$2x^2 = 24$ isolate x^2

$x^2 = 12$

$x = \pm\sqrt{12}$ square root property

$x = \pm\sqrt{4 \cdot 3}$

$\boxed{x = \pm 2\sqrt{3}}$ simplify radical

(6) $(2x-5)^2 = -16$ The square $(2x-5)^2$ is already isolated.

$2x - 5 = \pm\sqrt{-16}$ square root property

$2x - 5 = \pm\sqrt{-1}\sqrt{16}$ simplify radical

$2x - 5 = \pm 4i$

$2x = 5 \pm 4i$ isolate x

$x = \frac{5}{2} \pm \frac{4}{2}i$ divide each to get $a+bi$ form

$\boxed{x = \frac{5}{2} \pm 2i}$ simplify

Math 70

7 $3p^2 - 6p = 4$

CANNOT EVER USE CTS WHEN LEADING COEFFICIENT $\neq 1$

Method I: Divide all terms by leading coefficient, even this creates fractions throughout.

$p^2 - 2p = \frac{4}{3}$

To complete the square, take middle coefficient, divide by 2, and square result.

$\# = \frac{-2}{2} = -1$ ← This value is factor value

$\#^2 = (-1)^2 = 1$ ← Add this to both sides

$p^2 - 2p + 1 = \frac{4}{3} + 1$ ← add $\#^2$ to both sides

$(p-1)^2 = \frac{7}{3}$ ← use factor # to write perfect sq. on LHS

$p - 1 = \pm \frac{\sqrt{7} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$ ← rationalize

$\boxed{p = 1 \pm \frac{\sqrt{21}}{3}}$ or $\boxed{\frac{3 \pm \sqrt{21}}{3}}$

There are two solutions $1 + \frac{\sqrt{21}}{3}$ and $1 - \frac{\sqrt{21}}{3}$

(or $\frac{3+\sqrt{21}}{3}$ and $\frac{3-\sqrt{21}}{3}$).

$\sqrt{21}$ is an irrational number, but real. (no i)

So there are $\boxed{\text{2 solutions irrational, real}}$

Method 2: Factor Out Leading Coefficient

Process for Solving by Completing the Square

step 1: Arrange equation with all variables on LHS and constant on RHS.

7 $\quad 3p^2 - 6p \quad = 4$

step 2: Factor out the leading coefficient (or divide both sides by it)

$3p^2 - 6p \quad = 4$

$3(p^2 - 2p \quad) = 4$

$\frac{3p^2}{3} - \frac{6p}{3} \quad = \frac{4}{3}$

$p^2 - 2p \quad = \frac{4}{3}$

step 3: Find the coefficient of the degree 1 term and divide it by 2.
This number will be used in step 6 to write the perfect square.

$\# = \frac{-2}{2} = -1$

step 4: Square the result from step 3.

$\#^2 = (-1)^2 = 1$

step 5: Add the result from step 4 inside parentheses on LHS of equation.
Mentally distribute and add result to RHS of equation.

$\underbrace{3(p^2 - 2p + 1)}_{3 \cdot 1 = 3} = 4 + 3$

$3(p^2 - 2p + 1) = 7$

step 6: Use the result in step 3 to factor the quantity in parentheses to form a perfect square.

$3(p-1)^2 = 7$

step 7: Solve using the square root property

$(p-1)^2 = \frac{7}{3}$

$p - 1 = \pm\sqrt{\frac{7}{3}}$

$p - 1 = \pm\frac{\sqrt{7}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$

$p - 1 = \frac{\pm\sqrt{21}}{3}$

$\boxed{p = 1 \pm \frac{\sqrt{21}}{3}}$

Math 70

7 cont

Check $p = \frac{3+\sqrt{21}}{3}$ by evaluating by hand. Easier $1 + \frac{\sqrt{21}}{3}$.

$$3p^2 - 6p = 4$$

substitute for p.

$$3\left(1 + \frac{\sqrt{21}}{3}\right)^2 - 6\left(1 + \frac{\sqrt{21}}{3}\right) = 4$$

$$3\left(1 + \frac{\sqrt{21}}{3}\right)\left(1 + \frac{\sqrt{21}}{3}\right) - 6 - 2\sqrt{21} = 4$$

$$3\left(1 + 2 \cdot 1 \cdot \frac{\sqrt{21}}{3} + \frac{21}{9}\right) - 6 - 2\sqrt{21} = 4$$

$$3 + \frac{6\sqrt{21}}{3} + \frac{21}{3} - 6 - 2\sqrt{21} = 4$$

$$3 + 2\sqrt{21} + 7 - 6 - 2\sqrt{21} = 4$$

$$4 = 4$$ ☺

Check $p = \frac{3-\sqrt{21}}{3} = 1 - \frac{\sqrt{21}}{3}$ by hand

$$3\left(1 - \frac{\sqrt{21}}{3}\right)^2 - 6\left(1 - \frac{\sqrt{21}}{3}\right) = 4$$

$$3\left(1 - \frac{\sqrt{21}}{3}\right)\left(1 - \frac{\sqrt{21}}{3}\right) - 6 + 2\sqrt{21} = 4$$

$$3\left(1 - 2 \cdot \frac{\sqrt{21}}{3} + \frac{21}{9}\right) - 6 + 2\sqrt{21} = 4$$

$$3 - 2\sqrt{21} + 7 - 6 + 2\sqrt{21} = 4$$

$$4 = 4$$ ☺

Both $p = 1 + \frac{\sqrt{21}}{3}$ and $p = 1 - \frac{\sqrt{21}}{3}$ are solutions of $3p^2 - 6p = 4$.

Math 70

7 con't

Check $p = \dfrac{3+\sqrt{21}}{3}$ by evaluating function by GC:

In GC $(3+\sqrt{\ }(21))/3$ $\boxed{\text{STO}\blacktriangleright}$ $\boxed{\text{ALPHA}}$ $\boxed{8}$ $\boxed{\text{ENTER}}$ ← This stores all the decimal places in memory location P.

"P" location

2.527525232

In GC $\boxed{\text{Y=}}$ $Y_1 = 3x^2 - 6x - 4$

In GC $\boxed{\text{VARS}}$ $\boxed{>}$

To $\boxed{\text{Y-VARS}}$ MENU

$\boxed{1}$ Function

$\boxed{1}$ Y_1

In screen

Y_1

Type $\boxed{(}$ $\underbrace{\boxed{\text{ALPHA}}\ \boxed{8}}_{\text{"P"}}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

This evaluates Y_1 when X = value stored in memory location P.

$Y_1(P)$

0

This value of P <u>IS</u> a solution of Y_1.

$\boxed{\text{2nd}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ $\boxed{\text{ENTER}}$ ⇒ gives back what we typed 2 entries ago.

Use $\boxed{\blacktriangleleft}$ to move on top of +, then type $\boxed{-}$, and press $\boxed{\text{ENTER}}$

$(3-\sqrt{\ }(21))/3 \to P$ stores $-.5275252317$ in location P.

$\boxed{\text{2nd}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{2nd}}$ $\boxed{\text{ENTER}}$ ⇒ gives back what we typed 2 entries ago

$Y_1(P)$ ← Now it's using the new value stored in memory location P.

$\boxed{\text{ENTER}}$

$(3-\sqrt{\ }(21))/3 \to P$

$-.5275252317$

$Y_1(P)$

0

$Y_1(P)$ means

$$3\left(\frac{3-\sqrt{21}}{3}\right)^2 - 6\left(\frac{3-\sqrt{21}}{3}\right) - 4 = 0$$

confirming that $p = \dfrac{3-\sqrt{21}}{3}$ is also a solution.

<u>Note</u>: You can also type in the number instead of using memory location values.

Method 1a: CTS by Factoring out leading coefficient

8 Extra

$3x^2 - 9x + 8 = 0$ (2 complex)

$3(x^2 - 3x \quad) = -8$

$\#^2 = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$

$3\left(x^2 - 3x + \frac{9}{4}\right) = -8 + 3 \cdot \frac{9}{4}$

HINT: Use GC and >frac

$3\left(x - \frac{3}{2}\right)^2 = \frac{-5}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{-5}{12}$

$x - \frac{3}{2} = \pm\sqrt{\frac{-5}{12}}$

$x = \frac{3}{2} \pm \frac{i\sqrt{5} \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}}$

$\boxed{x = \frac{3}{2} \pm \frac{i\sqrt{15}}{6}}$

$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(3)(8)}}{2(3)}$

$x = \frac{9}{6} \pm \frac{\sqrt{-15}}{6}$

$x = \frac{3}{2} \pm \frac{i\sqrt{15}}{6}$ 2 complex solutions

Method 1B
CTS: divide by leading coef.

$3x^2 - 9x + 8 = 0$ (2 complex)

$\frac{3x^2}{3} - \frac{9x}{3} = \frac{-8}{3}$

$x^2 - 3x = \frac{-8}{3}$

CTS $\begin{cases} \# = -\frac{3}{2} & \leftarrow \text{use in factor} \\ \#^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4} & \leftarrow \text{add to both sides} \end{cases}$

$x^2 - 3x + \frac{9}{4} = -\frac{8}{3} + \frac{9}{4}$

MATH >frac is your friend!!

$\left(x - \frac{3}{2}\right)^2 = \frac{-5}{12}$

Simplify $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

$x - \frac{3}{2} = \frac{\pm i\sqrt{5}}{2\sqrt{3}}$

$x = \frac{3}{2} \pm \frac{i\sqrt{5} \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} \Rightarrow \boxed{x = \frac{3}{2} \pm \frac{i\sqrt{15}}{6}}$

2 complex solutions